A method to overcome the problem of 'slow' sensors

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Intro

In general sensors have a certain inertia, they follow a sudden change of the measured quantity with a delay. As a result the signal received appears to be smoothed in time. The sensor is slow in the sense that its response time is larger than the shortest time scale we want to resolve. In many cases it is desirable to have a faster response of the sensor. Since the higher frequency parts of the signal are damped but not erased it is possible to reconstruct the original signal.

We present a method based on digital filters. This method can be applied in realtime to the incoming dataflow. It is possible to adapt the filter to the characteristics of the sensor and the noise in the signal. Possible applications are REA systems, radiosoundings etc.

Example

We measured Temperature (T) and Humidity (rH) on the cable cars at Zugspitze mountain, Germany (see Poster EGU2007-A-10161). Every ride of the cable cars a profile was measured. Due to the inertia of the sensors the measured values of the ride up differ from the ride down resulting in a hysteresis like effect (see fig. 1). We use the method presented here to reconstruct real profiles.

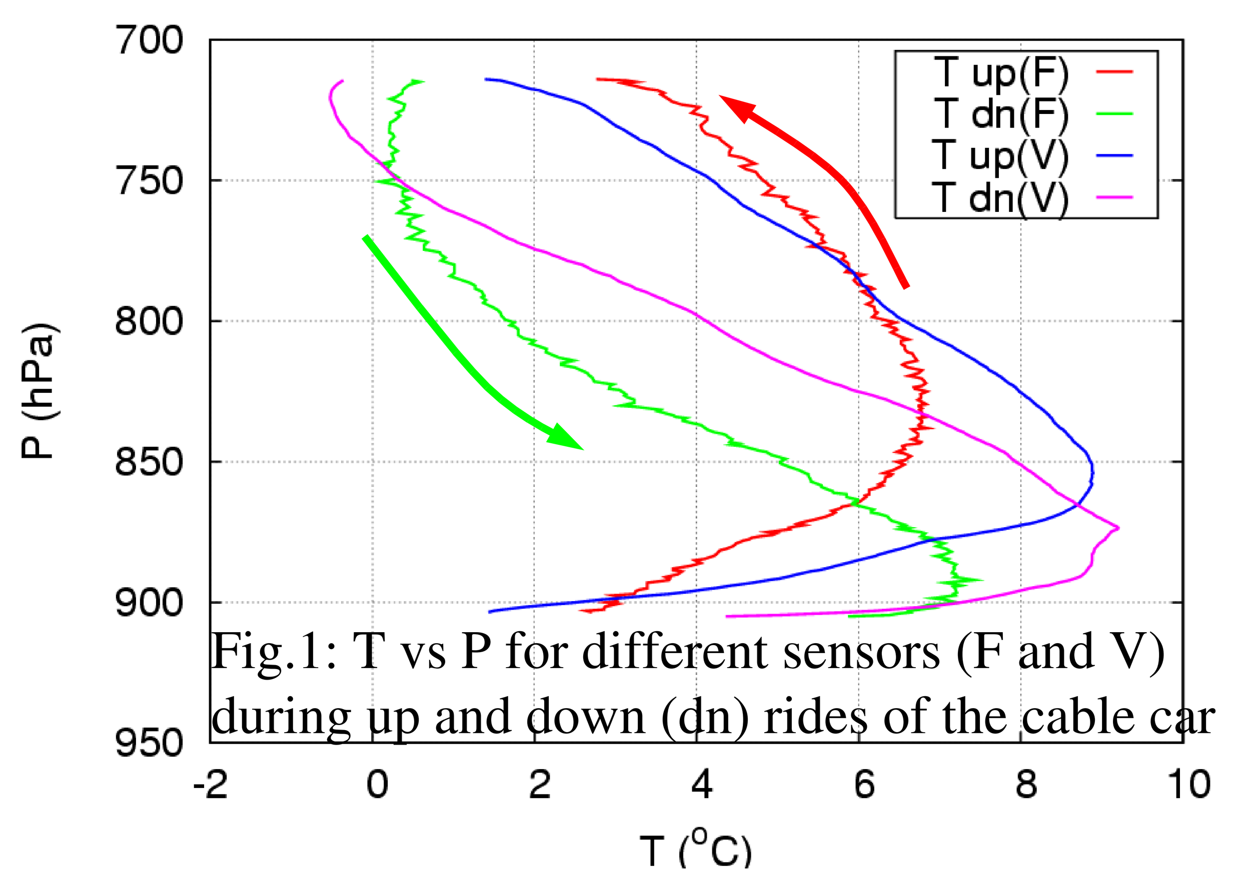


Fig.1: T vs P for different sensors (F and V) during up and down (dn) rides of the cable car

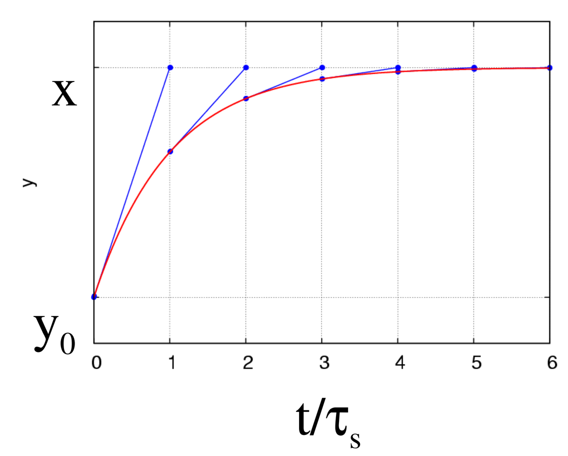
Numerics

$$\frac{dy}{dt}=-\frac{1}{\tau_s}\cdot(y-x) \quad [1]$$

y = sensor value
x = ambient value
τ_s = time constant of the sensor

special solution for $x = const.$:

$$y(t)=x+(y_0-x)\cdot\exp(-t/\tau_s) \quad [2]$$



A tangent at every point of *y(t)* reaches after $\Delta t=\tau_s$ the ambiance value x

A Laplace transformation gives the general solution of eq.[1]:

$$y(t)=\frac{1}{\tau_s}\int_{-\infty}^{t} x(s)\cdot\exp\left(\frac{s-t}{\tau_s}\right) ds \quad [3]$$

This is a low pass filter and is aproximated very well by a *digital recursive Filter*:

$$y(t)\simeq 坬 q_s\cdot y(t-t)+(1-q_s)\cdot x(t) \quad [4]$$
$$q_s= 坬 \exp(-t/\tau_s)$$

Equation [4] can be solved for the ambient value x giving the equation

$$x(t)=\frac{y(t)-q_s\cdot y(t-t)}{1-q_s} \quad [5]$$

With this *digital recursive filter* the ambient values *x(t)* can be ***reconstructed*** from only two subsequently measured values *y(t)* and *y(t −Δt)*.

Noise

At higher frequencies noise becomes a considerable part of the signal and is enhanced in the reconstruction by equation [5]. This noise can be reduced by applying a smoothing filter to the measured signal prior to the reconstruction. For the smoothing filter we use a recursive filter as given by equation [4] with a time constant τ_f.

Equation [3] (and in good approximation also eq. [4]) corresponds to a filter function

$$F_{s/f}(\omega)=\frac{1}{1+(\omega\cdot\tau_{s/f})^2} \quad [6]$$

The sensor and the subsequent smoothing are equivalent to applying a filter function $F_{fs}= F_s\cdot F_f$ to *x(t)*. The filter function corresponding to the reconstruction equation [5] is with good aproximation $F_x = 1+(\omega\cdot\tau_x)^2$.

If we reconstruct the signal we have to use a time constant τ_x which represents the combined filter F_{fs}. A suitable choice is a τ_x such that F_x^{-1} has the same edge frequency $\omega_{½}$ definded by $F_x(\omega_{½}) = ½$ as F_{fs}. Figure 2 shows the resulting filter functions for the presented example.

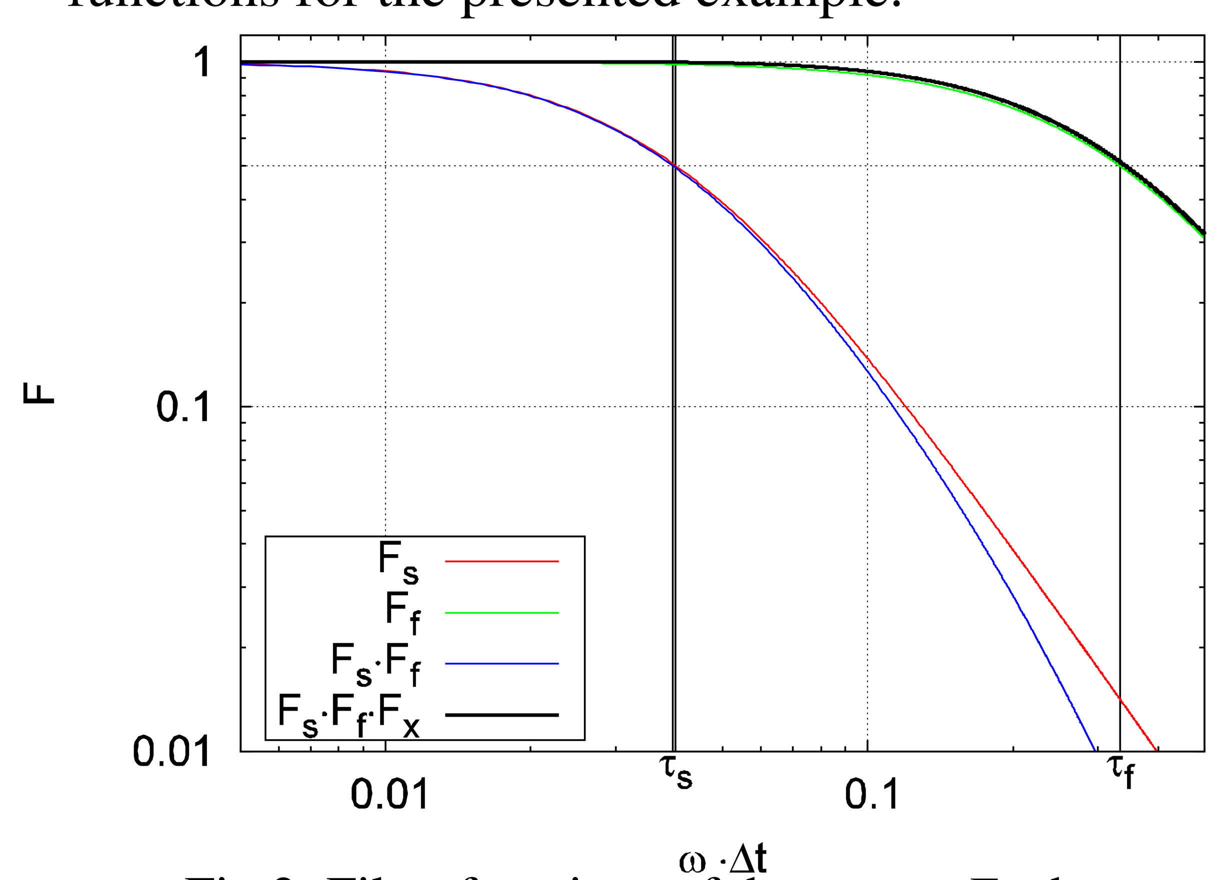


Fig.2: Filter functions of the sensor F_s, the smoothing filter F_f , their combination $F_s\cdot F_f$ and the final filter $F_s\cdot F_f\cdot F_x$

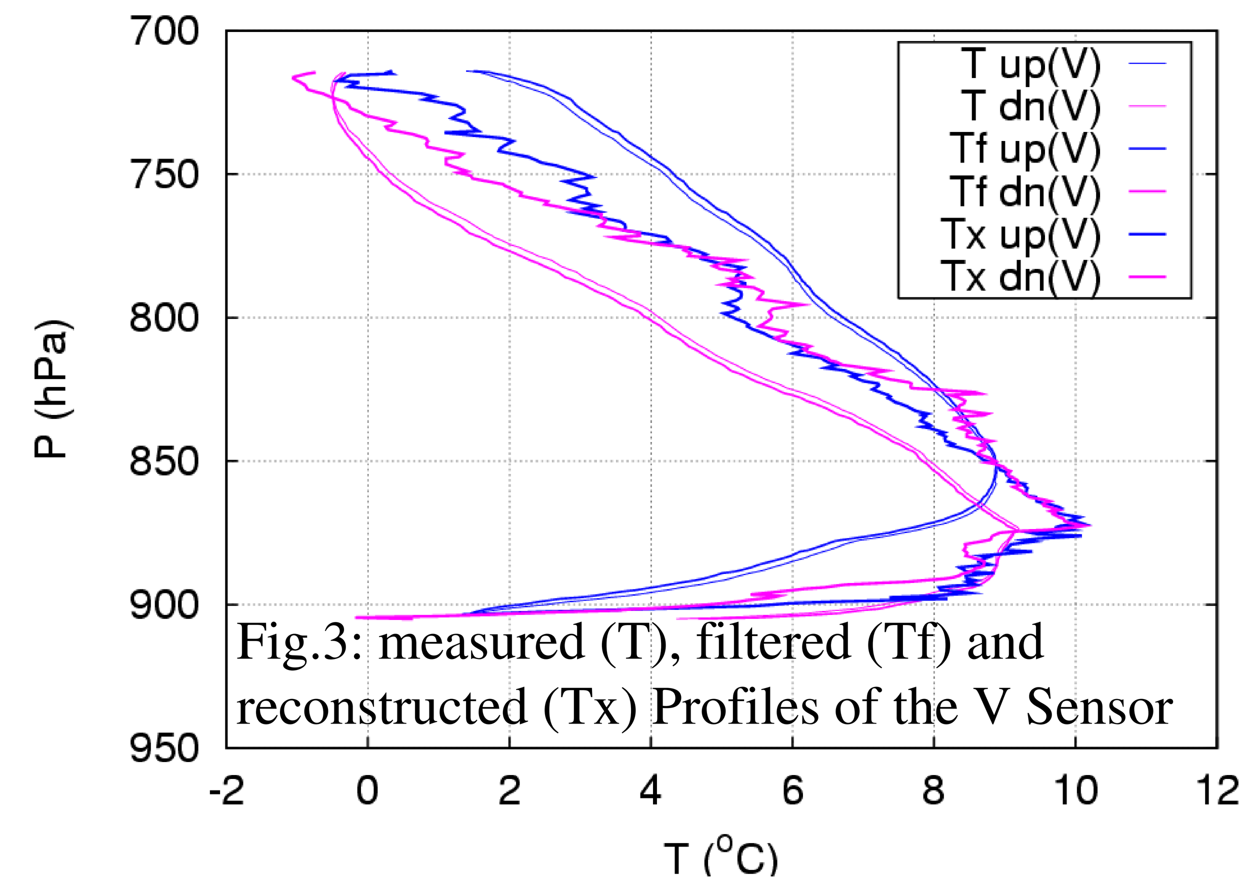


Fig.3: measured (T), filtered (Tf) and reconstructed (Tx) Profiles of the V Sensor

Application

Figure 3 shows the result for the temperature profiles of the Vaisala Sensor (V) shown in Figure 1. The original measured values were smoothed with the filter according to equation [4] and a time constant $\tau_f = 6s$ (i.e. the sensor was made artificially slower). Reconstruction of the ambient Temperature was made using equation [5] asuming $\tau_s = 50s$ and thus $\tau_x = 50.7s$.

Althoug the measured profiles seem to be rather smooth the reconstructed profile shows obvious noise.

Despite this noise there is general agreement between the two reconstructed Profiles indicating that the reconstruction is a good approximation to reality.

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