Signal Analysis Doppler-SODAR

MODOS
DSDR3x7
DSDPA.90-24/-64
PCS-2000/-24/-64

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1 General Remarks

As the received signal of the backscattered acoustic pulse can be quite small compared to noise contribution from the environment the signal analysis must be able to reduce the amount of noisy contributions significantly to allow a reliable determination of the Doppler shifted frequency. Electronic features like narrow band filters with matched bandwidth (typ. 100-200 Hz) will decrease white noise contributions only outside the frequency band of the acoustic signal in order to prevent the saturation of receiver units. Further signal improvement must be done by spectral analysis. Two ways of signal analysis are possible:

- Spectral analysis of individual pulses followed by estimation algorithms for the expected value of the frequency shift (consensus, Weber-Wuertz, Cluster, etc.). Such algorithms use acceptance thresholds for each pulse (minimum signal intensity, significance of a peak, etc.) to validate the individual signal and further acceptance thresholds to derive a representative value from the total ensemble of individual values collected for periods of 10 - 30 minutes.
- Determination of mean spectra of signal and noise signals during a corresponding averaging interval followed by spectral analysis and noise subtraction from the signal spectra. This approach allows a determination of the uncertainties in the noise measurements and a validation by comparing the statistics of the mean spectra for noise and signal.

As the background noise at most sites can change rapidly a noise reduction for individual pulses is often not essential due to the statistical uncertainties and instationarity. Therefore, the analysis of individual pulses is only sufficient in conditions when the background noise is low. In order to allow reliable measurements also at noisy sites the evaluation of mean spectra is superior as the uncertainties of the statistical properties will decrease by a factor of $\sqrt{n}$, with n as the number of acoustic pulses. Furthermore reliable objective acceptance criteria can be defined.

A disadvantage for the method of mean spectra results from the dependance of the signal on the atmospheric turbulence which varies strongly within minutes (for ex. in convective conditions). As updrafting air masses are correlated with higher signal intensity an overall shift in the mean spectra will be caused. But as all three independant measured radial components are influenced similar, the finally evaluated wind data are hardly affected. It should be noted that the individual pulse analysis is influenced by this effect as well because the probability for a low signal quality and for a rejection of an individual spectra is higher in downdraft than in updraft air masses.

It is the experience of the manufactuerer that the mean spectra evaluation is the more reliable method in noisy conditions, the individual analysis of spectra increases the data availability in upper heights in good measuring conditons. METEK SODARs will routinely follow the mean spectra analysis in the SODARs but allows simultaneously the output of the instantaneous data. So the user will get reliable data in a sufficient height range online but can perform a careful off-line analysis in order to increase the height coverage using the instananeous data.

Please note: there are no internal low pass filter algorithms or oversampling processes involved as such methods could possibly falsify data in weather conditions of strong temporaral variation or vertical shear. Using the „sodar.grafik" program package a smoothing routine is available.

2 Determination of Radial Components

The Fast-Fourier-Transformation performed on the received signal of each individual pulse computes for each height range 32 complex numbers for the amplitude. These spectral intensities are summed up for a typical number of 20 - 60 pulses which yields mean spectra for each height range consisting of the 32 averaged spectral intensities.

(2.1) E_n $(n = 1...32)$ with the signal received from an arbitrary height range

In addition to the reception of the backscattered acoustic pulses two independant noise measurements are performed shortly before the acoustic pulses are emitted. The observed noise signals are FFT-analyzed as well so for each pulse two noise power spectra are computed which are summed up according the method with the signal spectra:

(2.2) $R1_n$ $(n = 1...32)$

(2.3) $R2_n$ $(n = 1...32)$

Now the averaged intensities of and the max. deviation between the spectra of noise measurement 1 and 2 are computed for each line (the max. deviation is used for the internal plausibility checks). The averaged noise figures are subtracted from the received signal which gives the residual power spectra in each height:

(2.4) $S_n = (E_n - (R1n + R2n) / 2)$ $(n = 1...32)$

The derived power spectrum is approximated by a Gaussian shaped model function

(2.5) $$P = \frac{pwr \cdot \Delta f}{sig \cdot \sqrt{2 \cdot p}} \cdot e^{-\frac{1}{2} \bullet \left(\frac{frq - f}{sig} \right)^2}$$

with

Δf = Frequency resolution

Δh = Height resolution

c = Speed of sound

and using the parameter pwr, frq, sig to approximate the function curve using a least square fit ($Df = c/ 2Dh$ is the frequency resolution of the FFT, Dh is the height resolution).

The sound of speed is computed from the temperature measured by the PT100-probe according to:

(2.6) $c\,[m/s] = 20.05 \times \sqrt{T\,[K]}$

The sound of speed is used for the determination of the frequency shift and for the height ranging.

The sequential numbering of the *n* FFT-Lines can be seen as a frequency shift according to

(2.7) $f_n = (n-16)\,\Delta f$

For the approximation of the power spectra only the part showing the highest intensities is used, i.e. only the lines within a certain area neighbored to the main peak are considered. This reduces the sensitivity to broad band noise contribution. The limit of this area is determined by a threshold of 25 % of the maximum peak which corresponds to a 6 dB decrease from the max. signal intensity. So a number of n spectral lines is defined within such 6 dB area which are used to compute the regression between model function and power spectra. The parameter *pwr, frq* und *sig* are called moments of the density function.

pwr -also called „zero“ moment- corresponds to the area below the gaussian shaped curve which is the total power of the signal. *frq* as the 1. moment is the center of gravity on the frequency axis, it can be seen as the most likely value for the frequency shift of the signal. The parameter *sig* is identifed as the 2 moment which is the standard deviation of the distribution of the intensities, it corresponds to the width of the spectrum and is related to standard deviation of the frequency shift.

For mathematical reasons logarithmic values of the power spectra are used for further evaluation. Doing this the gaussian-shaped function is transformed to a parabolic function which can be handled easily. The parameter are derived from the spectra as follows:

pwr : Signal power in the function’s maximum ⇒ back scattered signal intensity
frq : Frequency shift with the maximum signal intensity ⇒ Doppler-Shift of the main radial component
sig : Curvature of the parabolic function (Variance of the radial components)

$$frq = -0.5\frac{D1}{D2}$$

$$sig = -0.5\frac{D}{D2}$$

(2.8)

$$pwr = sig\sqrt{2p}\cdot\exp^{\left(\frac{D2}{D}frq^2+\frac{D1}{D}frq+\frac{D0}{D}\right)}$$

according to

$$f(a,b,c) = \sum_{i=1}^{n}\left(y_i - ax_i^2 - bx_i - c\right)^2$$

$$= \sum_{i=1}^{n} y_i^2 - 2\sum_{i=1}^{n} y_i ax_i^2 - 2\sum_{i=1}^{n} y_i bx_i^2 - 2\sum_{i=1}^{n} cy_i + 2\sum_{i=1}^{n} abx_i^3 + 2\sum_{i=1}^{n} acx_i^2 + \sum_{i=1}^{n} b^2x_i^2 + 2\sum_{i=1}^{n} bcx_i^2 + \sum_{i=1}^{n} a^2x_i^4 + \sum_{i=1}^{n} c^2$$

$$f(a,b,c) \quad \rightarrow \quad \min$$

$$\frac{\partial f}{\partial a} = -2\sum_{i=1}^{n} y_i x_i^2 + 2\sum_{i=1}^{n} bx_i^3 + 2\sum_{i=1}^{n} cx_i^2 + 2\sum_{i=1}^{n} ax_i^4 \overset{!}{=} 0$$

$$\Rightarrow \quad a\sum_{i=1}^{n} x_i^4 + b\sum_{i=1}^{n} x_i^3 + c\sum_{i=1}^{n} x_i^2 \overset{!}{=} \sum_{i=1}^{n} y_i x_i^2$$

$$\frac{\partial f}{\partial b} = -2\sum_{i=1}^{n} y_i x_i + 2\sum_{i=1}^{n} ax_i^3 + 2\sum_{i=1}^{n} bx_i^2 + 2\sum_{i=1}^{n} cx_i \overset{!}{=} 0$$

$$\Rightarrow \quad a\sum_{i=1}^{n} x_i^3 + b\sum_{i=1}^{n} x_i^2 + c\sum_{i=1}^{n} x_i \overset{!}{=} \sum_{i=1}^{n} y_i x_i$$

$$\frac{\partial f}{\partial c} = -2\sum_{i=1}^{n} y_i + 2\sum_{i=1}^{n} ax_i^2 + 2\sum_{i=1}^{n} bx_i + 2\sum_{i=1}^{n} c \overset{!}{=} 0$$

$$\Rightarrow \quad a\sum_{i=1}^{n} x_i + b\sum_{i=1}^{n} x_i + c\cdot n \overset{!}{=} \sum_{i=1}^{n} y_i x_i$$

$$D = \begin{vmatrix} \sum_{i=1}^{n} x_i^4 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i & n \end{vmatrix}$$

$$D_0 = \begin{vmatrix} \sum_{i=1}^{n} x_i^4 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} y_i x_i^2 \\ \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} y_i x_i \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i \end{vmatrix} \qquad D_1 = \begin{vmatrix} \sum_{i=1}^{n} x_i^4 & \sum_{i=1}^{n} y_i x_i^2 & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} y_i x_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} y_i & n \end{vmatrix} \qquad D_2 = \begin{vmatrix} \sum_{i=1}^{n} y_i x_i^2 & \sum_{i=1}^{n} x_i^3 & \sum_{i=1}^{n} x_i^2 \\ \sum_{i=1}^{n} y_i x_i & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} y_i & \sum_{i=1}^{n} x_i & n \end{vmatrix}$$

$$D = n \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^4 + 2 \cdot \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^3 - \left(\sum_{i=1}^{n} x_i^2 \right)^3 - \left(\sum_{i=1}^{n} x_i \right)^2 \cdot \sum_{i=1}^{n} x_i^4 - n \cdot \left(\sum_{i=1}^{n} x_i^3 \right)^2$$

$$D_0 = \sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^4 + \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i^2 y_i \cdot \sum_{i=1}^{n} x_i^3 + \sum_{i=1}^{n} x_i y_i \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^3 - \left(\sum_{i=1}^{n} x_i^2 \right)^2 \cdot \sum_{i=1}^{n} x_i^2 y_i - \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i y_i \cdot \sum_{i=1}^{n} x_i^4 - \sum_{i=1}^{n} y_i \cdot \left(\sum_{i=1}^{n} x_i^3 \right)^2$$

$$D_1 = n \cdot \sum_{i=1}^{n} x_i y_i \cdot \sum_{i=1}^{n} x_i^4 + \sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^3 + \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^2 y_i - \sum_{i=1}^{n} x_i y_i \cdot \left(\sum_{i=1}^{n} x_i^2 \right)^2 - \sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i^4 - n \cdot \sum_{i=1}^{n} x_i^2 y_i \cdot \sum_{i=1}^{n} x_i^3$$

$$D_2 = n \cdot \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} x_i^2 y_i + \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i y_i \cdot \sum_{i=1}^{n} x_i^2 + \sum_{i=1}^{n} y_i \cdot \sum_{i=1}^{n} x_i \cdot \sum_{i=1}^{n} x_i^3 - \sum_{i=1}^{n} y_i \cdot \left(\sum_{i=1}^{n} x_i^2 \right)^2 - \left(\sum_{i=1}^{n} x_i \right)^2 \cdot \sum_{i=1}^{n} x_i^2 y_i - n \cdot \sum_{i=1}^{n} x_i y_i \cdot \sum_{i=1}^{n} x_i^3$$

3 Determination of Wind Speed and Wind Direction

The radial wind components is determined from the shift between the frequencies of the emitted acoustic pulse and the received -noise compensated- signal. Using 3 or 5 antenna beams a corresponding number of radial components is derived. The radial components VR_i corresponds to the parallel components of the wind vector when projected to the antenna beams A_i

(3.1) $$VR_i = c \cdot \frac{f_e - f_s}{f_e + f_s}$$

with f_s = Emitted acoustic frequency
f_e = Received acoustic frequency
c = Speed of sound
i = Index indicating the antenna beam , i = 1...3 or i = 1..5

To determine wind speed and wind direction 3 independently measured radial components VR_i, i=1..3 are required. So any three antenna beams must be orientated in a non-complanar way. A typical configuration of the antenna beams is:

$a_2 = a_1 + 90°$
a_3 = arbitrary
$j_1 = j_2 = 20°$
$j_3 = 0°$

a_1, a_2, a_3 = Azimuth of antenna beams A1, A2, A3
j_1, j_2, j_3 = Zenith of antenna beams A1, A2, A3

Wind speed (V) and wind direction (D) are derived from the radial components according to:

(3.2) $$V = \frac{\sqrt{Z^2 + N^2}}{\sin j}$$

and

(3.3)
$$D = \arctan |Z/N| + a_1 \qquad Z = 0, N = 0$$
$$D = 180° - \arctan |Z/N| + a_1 \qquad Z = 0, N = 0$$
$$D = 180° + \arctan |Z/N| + a_1 \qquad Z = 0, N = 0$$
$$D = 360° - \arctan |Z/N| + a_1 \qquad Z = 0, N = 0$$

and

(3.4) $$Z = \frac{VR_2 + VR_3 \cos j\, (\cos(a_2 - a_1) - 1) - VR_1 \cos(a_2 - a_1)}{\sin(a_2 - a_1)}$$

and

(3.5) $N = (VR_1 - VR_3 \cos j)$

and

VR_1, VR_2, VR_3	=	Radial components antenna beams A1, A2, A3
j	=	Zenith angle A1and A2
a_1, a_2	=	Azimuth angle A1 and A2

4 Accuracies

To specify accuracies for the measured variables a common way is a comparison to mast instrumentation. However certain limitations exit for this method. If the SODAR antenna is located close to the mast (in order to minimize the separation of the measuring volumes) fixed echo contribution will deteriorate the signal. Furthermore, the in-situ mast sensor will measure within a volume of some decimeters compared to the SODAR with volumes of some meters or even decameters. For that reasons theoretical analysis and numerical simulation is a good alternative.

The fundamental relation for SODAR measurements is

$$(4.1) \quad VR = c \cdot \frac{f_e - f_s}{f_e + f_s}$$

or in good approximation

$$(4.1a) \quad VR = c \cdot \frac{f_e - f_s}{2 \cdot f_s} = \frac{c \cdot \Delta f}{2 \cdot f_s}$$

with

- VR = Wind component projected parallel to the antenna beam (radial component)
- c = Speed of sound
- f_e = Backscattered frequency
- f_s = Emitted frequency
- Δf = $f_e - f_s$ = Doppler shift

As the speed of sound is not known within the measuring volume the temperature must be measured near the antenna at the ground. The inaccuracy that might occur due to strong temperature gradients must is considered as:

$$(4.2) \quad \frac{\Delta c}{c} = \sqrt{1 + \frac{\Delta T}{T}} - 1 = \frac{1}{2} \cdot \frac{\Delta T}{T}$$

with
- Δc : Deviation in speed of sound
- ΔT : Deviation in temperature
- T : Temperature (~ 300 K)

From that the relative error in speed of sound is about 1 % for 6 °K which is a typical value for SODAR measurements within the first some hundred meters. In case of strong inversions the deviation will increase but should never exceed +- 3 %.

The analysis of the frequencies in equation 3.1 can be performed with very high accuracy but deviations result from the unavoidable statistical uncertainties in the spectral analysis due to the limited (and sometimes rather low) signal/noise ratio in the spectra. Furthermore the signal/noise ratio can change very rapidly according to the variation of the background noise. Therefore, automatic operating systems must automatically evaluate the signal quality and must reject or at least mark any data having an uncertainty higher than the specified system accuracy.

4.1 Relation between Unaccuracies and Signal/Noise ratio

The SODAR system emits pulses with a frequency f_s and duration t_s, which will propagate with the speed of sound. The time variation of the signals can be seen as a wave function s(t). A scattering structure g in the distance x to the antenna will backscatter a small portion arriving at the antenna with a delay time of t = 2x/c. The amplitude e(t) of the received scatter signal which originates from this specific scattering structure can be defined by

$$de(t) = s(t - \frac{2x}{c})g(x)dx \qquad (4.3)$$

The received signal of a structure which is distributed within a whole volume will be derived from the integral form

$$e(t) = \int_{-\infty}^{+\infty} s(t - \frac{2x}{c})g(x)dx \qquad (4.4)$$

Substituting x => 2x/c leads to a convolution integral

$$e(t) = \frac{c}{2}\int_{-\infty}^{+\infty} s(t - \frac{2x}{c})g(\frac{2x}{c})d(\frac{2x}{c}) \qquad (4.5)$$

According to the mathematics of Fourier transformations this corresponds to a simple multiplication of the functions s(t-2x/c) and g(2x/c) in the frequency domain. Therefore, the spectra of the received signal can be written as

(4.6) $E(f) = S(f) * G(f)$

having G(f) proportional to the wave number spectrum of the scattering turbulence structure. The wave number is derived from

(4.7) $k = 2\, p\, f / c_0$.

The power spectrum is derived from

(4.8) $P(f) = E(f) * E^*(f)$

using E*(f) as the complex part of E(f).

The SODAR system uses pulses of time duration t_s and a oscillating frequency f_s (fs >> 1/ t_s). As the band width (B_s) of the signal is about 1/ts is, the spectrum S(f) concentrates most of the energy at the frequency fs within an band of B_s = 1/ts. The turbulence spectrum depends only weakly on f, so it is assumed that G(f) = "constant". So E(f) is proportional to S(f).

As the height resolution ΔH is determined by the pulse length ts by ΔH = 1/2*c*ts, the spectra of the received signal shows a certain band width B_e=c/(2*ΔH) even if an idealized turbulence free atmosphere is involved (please note: such situation would be not very realistic as no signal would be scattered back !!).

As the frequencies are related to the radial components by the Doppler relation the width of the velocity spectra corresponds to

(4.9) $$\Delta Vr = \frac{B_e}{2fs} \cdot c = \frac{c^2}{4fs\, \Delta H}$$

Due to the limited width of the Doppler spectrum and the limited signal/noise ratio S/R the determination is always effected by an uncertainty, which is prescribed as the standard deviation of the radial component s(Vr). The variable s(Vr) is very sensitive to the used algorithms inside the SODAR processing units and is seen as a system specific feature.

4.2 Quantitative Determination of Unaccuracies

The emitted signal is a function of time, it can be written as

(4.10) $s(t) = \exp(-3t^2/2t_s^2) * \sin(2\ p\ fs * t)$ for $t < ts$
$s(t) = 0$ for $t > ts$

The spectrum of the received signal is derived for the time intervall t_s by a 32-point FFT having a weigthing function

(4.11) $w(t) = \exp(-3t^2/2t_s^2)$ for $t < ts/2$
$w(t) = 0$ for $t > ts/2$

By numerical simulation a probability P(f) (see (4.8)) was calculated using G(f) as the Fourier transformation of an arbitrary distribution with δ-shaped autocorrelation und variance 1.

Such signal spectrum was superpositioned a white noise signal of variable power N. A further white noise spectrum was subtracted which had the same power, but which originated from a additional individual calculation. So the „true" signal was superpostioned by a noise spectrum having the power „Zero" and the Variance „2*N". In such way the influence of environmental noise on the signal quality and the efficiency of the internal noise compensation was tested.

As long as individual spectra were analyzed the spectra showed a rather arbitrary shape. The expected Gaussian shape showed up when the number of averaged signals became significant, typ. > 40. This corresponds for a pulse repitition rate of 5 s to a 10 minutes averaging interval. The statistical uncertainties were decreased by a factor of 6.5 (√40) (See some samples in the annex).

So using mean spectra the approximation with a Gaussian shape function and the derivation of scattered power, radial component and standard deviation by means of the moments of this function is justified.

The unaccuracies in the determination of the 1. moment was also calculated by numerical simulation. For that the above mentioned calculation was repeated 100 times with independant arbitrary numbers.The uncertainty is derived by

(4.12) $$s_{\Delta f} = \frac{\sqrt{\sum_{i=1}^{100}(\Delta f_i - \overline{\Delta f})^2}}{10}$$

The result is shown in annex 1 showing the standard deviation in distance units of the FFT-lines. The calculation to velocity follows eq. (4.9). ΔVr (the width of the velocity spectrum) corresponds to the velocity of the distance between two FFT-lines.

In case of good S/R (i.e. low noise and high reflectivity) the standard deviation of the derived frequency s(Δf) moves to a values of 0.06 lines. In case of poor signal intensity it raises to a value of 10 which is an arbitrary distribution over all FFT-lines.

In Metek's SODAR systems DSDPA.90 and MODOS/DSDR3x7 all values showing S/R < 3dB are rejected. For a height resolution of ΔH = 20 m and an operating frequency of f_s = 1675 Hz this corresponds to an uncertainty in the radial components of

(4.13) $s(Vr) = DH * s(Df) = 0.1 \quad m/s$

In the same way the uncertainty in the derivation of the second moment can be derived:

(4.14) $$s_{sf} = \frac{\sqrt{\sum_{1}^{100}(s(fi) - \overline{s(fi)})^2}}{10}$$

In annex 2 the standard deviation is shown again in units of the FFT-lines, the acceptance threshold is set again to 3 dB. For S/R ≥ 3 dB the deviation is about 0.2 bins, which corresponds to 0,17 m/.

Further analysis yields the uncertainty of the measured wind velocity:

(4.15) $$s(V) = \frac{\sqrt{3}}{\sin j} \cdot s(Vr) = 0.5\ m/s$$

for j = Zenith angle = 20°

For the wind direction the unaccuracy is

(4.16) $$s(D) = \frac{3}{\sin j} \cdot \frac{1}{V} \cdot \frac{360}{2p} = 5.7\,°$$

for V = Wind velocity = 5 m/s

In our calculation the errors in the three antenna beams were assumed to be independent. In reality the deviations are partly correlated so the resulting error will be smaller in most conditions.

Annex 1-2